

# Transmission Network Restoration Considering AC Power Flow Constraints

Ali Arab, Amin Khodaei, *Senior Member, IEEE*, Suresh K. Khator, and Zhu Han, *Fellow, IEEE*

**Abstract**—This paper develops an efficient model for power system restoration after natural disasters based on AC power flow constraints. The objective is to derive the optimal restoration schedule in order to minimize the real power load interruptions in the post-disaster phase. The load criticality is represented by using the value of lost load which prioritizes the loads to be restored. A linear AC formulation is further proposed to allow the calculation of voltage angle and reactive power in the network, hence ensuring a more practical solution compared to DC power flow based models. Mixed-integer programming is used to formulate the proposed restoration model. Numerical analysis on the IEEE 118-bus test system demonstrates the effectiveness and the applicability of the proposed restoration model.

**Index Terms**—AC power flow, natural disaster, mixed-integer programming, power system restoration.

## NOMENCLATURE

### Indices:

$b$	Index for buses.
$i$	Index for generation units.
$l$	Index for transmission lines.
$t$	Index for time.

### Parameters:

$SL_l^{max}$	Apparent power flow capacity in line $l$ .
$b_l$	Susceptance of line $l$ .
$PD_{bt}$	Active load at bus $b$ at time $t$ .
$QD_{bt}$	Reactive load at bus $b$ at time $t$ .
$g_l$	Conductance of line $l$ .
$M$	Large positive constant.
$P_i^{max}$	Maximum active power generation of unit $i$ .
$P_i^{min}$	Minimum active power generation of unit $i$ .
$PL_l^{max}$	Maximum active power flow capacity of line $l$ .
$Q_i^{max}$	Maximum reactive power generation of unit $i$ .
$Q_i^{min}$	Minimum reactive power generation of unit $i$ .
$QL_l^{max}$	Maximum reactive power flow of line $l$ .

$R_{kt}$	Number of crew members allocated to component $k$ at time $t$ .
$R_k^{min}$	Number of hourly required crew to repair component $k$ at time $t$ .
$R_t^{max}$	Maximum available repair crew at time $t$ .
$S_i$	Apparent power capacity of generation unit $i$ .
$TTR_k$	Time to repair for component $k$ .
$V_b^{max}$	Maximum voltage magnitude at bus $b$ .
$V_b^{min}$	Minimum voltage magnitude at bus $b$ .
$\tilde{V}_{bt}$	Fixed voltage magnitude of bus $b$ at time $t$ .
$VOLL_{bt}$	Value of lost load at bus $b$ at time $t$ .
$\alpha_{ib}$	Element of unit $i$ and bus $b$ in generation-bus incidence matrix.
$\beta_{lb}$	Element of line $l$ and bus $b$ in line-bus incidence matrix.
$\gamma_k$	Angle of an ellipse.

### Variables:

$\widehat{\cos}_{nmt}$	An auxiliary variable for cosine of voltage angle difference between buses $n$ and $m$ at time $t$ .
$I_{it}$	Unit commitment variable for generation of unit $i$ at time $t$ ; 1 if committed, otherwise 0.
$LI_{bt}$	Active load interruption at bus $b$ at time $t$ .
$LL_{bt}$	Reactive power shortage at bus $b$ at time $t$ .
$n_{it}$	Auxiliary binary variable.
$P_{it}$	Active power generation of unit $i$ at time $t$ .
$PL_{lt}$	Active Power flow of line $l$ at time $t$ .
$Q_{it}$	Reactive power generation of unit $i$ at time $t$ .
$QL_{lt}$	Reactive power flow of line $l$ at time $t$ .
$u_{bt}$	Binary repair state of bus $b$ at time $t$ .
$V_{bt}$	Voltage magnitude of bus $b$ at time $t$ .
$v_{lt}$	Binary repair state of line $l$ at time $t$ .
$w_{lt}$	Binary outage state of line $l$ at time $t$ ; 0 if offline due to damage, otherwise 1.
$y_{it}$	Binary outage state of unit $i$ at time $t$ ; 0 if offline due to damage, otherwise 1.
$z_{bt}$	Binary outage state of bus $b$ at time $t$ ; 0 if offline due to damage, otherwise 1.
$\delta_{bt}$	Voltage angle of bus $b$ at time $t$ .

This work was partially supported by the U.S. National Science Foundation under grants CMMI-1434789 and CMMI-1434771, and Electric Power Analytics Consortium funded by CenterPoint Energy, Inc.

Ali Arab and Suresh K. Khator are with the Department of Industrial Engineering, University of Houston, Houston, TX 77204, USA (Email: aarab@uh.edu; skhator@uh.edu).

Amin Khodaei is with the Department of Electrical and Computer Engineering, University of Denver, Denver, CO 80208, USA (Email: amin.khodaei@du.edu).

Zhu Han is with the Department of Electrical and Computer Engineering, University of Houston, Houston, TX 77004, USA (Email: zhan2@uh.edu).

## I. INTRODUCTION

The increasing number of natural disasters and extreme weather events with devastating aftermaths have significantly impacted the electricity infrastructure and reliable supply of power to consumers in recent years. This issue calls for new research efforts in more accurate modeling of these events and development of efficient recovery schemes to minimize load interruptions. Among other factors, an accurate modeling of the power network is of ultimate importance to obtain a more practical recovery solution for the system.

Many of current power network models rely on a DC power flow approximate to determine system behavior. [1] examines the merits of different versions of DC models for various applications. Different categories of DC models, e.g., hot-start, cold-start, sparse, and sensitivity factor models are analyzed. The results show that the accuracy of DC power models in general must never be taken for granted. It has been shown in recent published studies that the accuracy of DC power flow model in circumstances other than the normal operating condition is under question. Therefore, it is imperative to utilize AC power flow models which are demonstrated to provide a more reliable solution for applications such as *restoration* planning [2]. However, the tradeoff between computational efficiency and exactness of obtained solutions for AC models remains an open problem in various applications. In this context, a linear-programming approximation of AC power flows is proposed in [2]. A linear relaxation of DC and AC models to obtain efficient models for transmission system planning is presented in [3]. Recent advances in convex relaxation of optimal power flow (OPF) problem for different power network models can be found in [4], [5].

In context of restoration, [6] studies the budgeted and the minimum weighted latency variants of the recovery problem of large-scale power outage due to a major disaster. The problems for the general case as well as trees and bipartite networks as the special case are studied. In [7], a mixed-integer program to model the recovery of the transmission networks damaged due to disasters is formulated. The model considers the repair crew constraints as well as the penalty cost of unserved loads to find the recovery schedule which minimizes the cost of power outage. [8] uses a mixed-integer programming framework for modeling the optimal supply restoration of the faulty power distribution systems. A two-step decomposition method is developed to derive the optimal configuration as well as the optimal switching sequence of the power distribution system. In [9], a general multi-objective linear-integer spatial optimization model for arcs and nodes restoration of disrupted networked infrastructure after disaster is presented. The proposed model addresses the tradeoff between maximization of the system flow and minimization of system cost. [10] proposes an integrated network design and scheduling problem for restoration of the interdependent civil infrastructure. The problem is formulated using integer programming, and is analyzed on realistic dataset of power infrastructure of the Lower Manhattan in New York City and

New Hanover County, North Carolina. The results indicate that the proposed model can be used for real-time as well as long-term restoration planning. In another study, [11] considers “*the last-mile restoration*” of power systems, i.e., how to schedule and allocate the routes to the fleets of repair crews to recover the damaged power system as quickly as possible. The power restoration and vehicle routing are decoupled to improve the computational efficiency of the model. The proposed model outperforms the models which are practiced in the field in terms of solution quality and scalability. This work is extended in [12] by applying the randomized adaptive vehicle decomposition technique in order to improve the scalability of the model for large-scale disaster restoration of the power networks with more than 24,000 components.

In this paper, we extend our previous work [13] and [14] on restoration planning to integrate AC power flow constraints. The outage and repair constraints associated with damaged components are modeled with a focus on the outage of the component or any of the connected substations. A linear AC formulation is proposed to maintain the computational efficiency of the proposed model. Mixed-integer programming is used to formulate the problem.

The rest of paper is organized as follows: the proposed restoration model is described in Section II and formulated in Section III. Numerical studies are provided in Section IV to exhibit the effectiveness of the proposed model when applied to a test power system. Finally, the conclusions are drawn in Section V.

## II. MODEL DESCRIPTION

Natural disasters, such as hurricanes, can potentially damage power system components including generation units, transmission lines, substations, as well as downstream distribution lines. We propose a post-disaster restoration model to be used by Transmission & Distribution (T&D) utility companies to schedule the restoration of damaged transmission and distribution infrastructure in coordination with generation units.

After the disaster, the utility company conducts a damage assessment by an aerial survey of the power network in affected areas as well as a ground check by inspectors. Damage assessment determines whether a component is damaged at all, and if damaged, estimates the mean time to repair (TTR) for the component. Each substation along with its downstream distribution lines are aggregated and considered as a single component. Hence, the time to repair for each substation is aggregated in our model. While generation units are part of a vertically integrated utility company, it is assumed in our model that each generation unit is responsible for repair operations of its damaged facilities. Therefore, each damaged generation unit will submit its repair schedule to the utility company to be used as an input for restoration scheduling of transmission and distribution infrastructure. We consider two states for each component: *damaged*, if the component is encountered major damage, thus it is offline and needs to be repaired to be restored; and *functional*, if it has not

been damaged at all, or minor damages have occurred and the component is able to function.

We consider an AC power flow model which has proven to provide more accurate solutions for power system restoration. Furthermore, we propose a polyhedral inner approximation method for linearizing the quadratic apparent power equation. Therefore, we propose a restoration model with fully linearized AC constraints to ensure practicality and computational efficiency. The model intends to minimize the customer load interruption cost considering the value of lost load (VOLL). The output of the proposed model includes the post-disaster restoration schedule, power dispatch, bus voltage angles, and transmission network configuration.

### III. PROBLEM FORMULATION

#### A. Objective Function

The objective of the proposed restoration model is to minimize the real load interruption cost as follows:

$$\min_{LI} \sum_t \sum_b VOLL_{bt} LI_{bt}, \quad (1)$$

where  $VOLL_{bt}$  reflects the criticality of the loads to be recovered. A higher value of lost load results in a faster recovery of the transmission lines and substations connected to that load.

#### B. Power Balance Equation

The active load interruption is considered as a negative load (i.e., a virtual generation) and is obtained from the active power balance equation as follows:

$$\sum_{i \in N_b} P_{it} + \sum_{l \in N_b} PL_{lt} + LI_{bt} = PD_{bt}, \quad \forall b, \forall t. \quad (2)$$

The active power balance equation ensures that the injected active power to a bus from connected transmission lines and generating units matches the load, and if not adequate, the load will be curtailed by the active load interruption variable.

Similarly, the reactive power demand at each bus has to be supplied through generation units and transmission lines. Therefore, the reactive power balance equation is

$$\sum_{i \in N_b} Q_{it} + \sum_{l \in N_b} QL_{lt} + LL_{bt} = QD_{bt}, \quad \forall b, \forall t, \quad (3)$$

where,  $N_b$  is the set of connected generation units and transmission lines to bus  $b$ . Note that in many cases the reactive power shortage can be handled locally using available reactive power compensators in distribution networks.

#### C. Generation Outage and Capacity Constraints

The generation units' active and reactive power generations are limited to their minimum and maximum capacities. Binary variables  $y_{it}$  and  $z_{bt}$  are defined to model outage of the generating unit and the connected substation.  $y_{it}$  is equal to 0 if the generation unit  $i$  is offline at time  $t$  due to damage from disaster; otherwise it is equal to 1. The active and reactive power generations in each unit  $i$  are bounded to damage state,

commitment state, and minimum and maximum generation capacity as

$$P_i^{min} y_{it} I_{it} \leq P_{it} \leq P_i^{max} y_{it} I_{it}, \quad \forall i, \forall t, \quad (4)$$

$$Q_i^{min} y_{it} I_{it} \leq Q_{it} \leq Q_i^{max} y_{it} I_{it}, \quad \forall i, \forall t. \quad (5)$$

The active and reactive power generation limits turn out to be nonlinear constraints. To linearize these constraints an auxiliary variable  $n_{it} = y_{it} I_{it}$  is defined to decompose the constraint (4) as follows:

$$P_i^{min} n_{it} \leq P_{it} \leq P_i^{max} n_{it}, \quad \forall i, \forall t, \quad (6)$$

$$n_{it} - y_{it} \leq 0, \quad \forall i, \forall t, \quad (7)$$

$$n_{it} - I_{it} \leq 0, \quad \forall i, \forall t, \quad (8)$$

$$-n_{it} + y_{it} + I_{it} \leq 1, \quad \forall i, \forall t, \quad (9)$$

$$n_{it} \geq 0, \quad \forall i, \forall t. \quad (10)$$

Constraint (5) can be linearized in the same manner. Furthermore,  $z_{bt}$  is used to represent the outage of substation  $b$  at time  $t$ . The outage of the generating unit or the associated substation are incorporated to model the active and reactive power generation capacity constraints to impose a zero generation when any of these components is on outage. Therefore,

$$-M \sum_b \alpha_{ib} z_{bt} \leq P_{it} \leq M \sum_b \alpha_{ib} z_{bt}, \quad \forall i, \forall t, \quad (11)$$

$$-M \sum_b \alpha_{ib} z_{bt} \leq Q_{it} \leq M \sum_b \alpha_{ib} z_{bt}, \quad \forall i, \forall t. \quad (12)$$

Generator capability curve is an important constraint which needs to be taken into account as follows:

$$P_{it}^2 + Q_{it}^2 \leq S_i^2, \quad \forall i, \forall t, \quad (13)$$

which is a quadratic equation of a semi ellipse (in fact, the power capability curve is a circle which is a special case of ellipse). Therefore, we can approximate this curve as a polygon or polyhedron by adding linear constraints. As shown in Fig. 1, by dividing the semi ellipse into  $k$  slices, each with angle  $\gamma_k$ , we can approximate the feasible region of the generator capability curve as a semi polygon (for the sake of illustration of the idea, we have considered the  $P_i^{max}$  and  $Q_i^{max}$  as the length of semi major and semi minor axis of the ellipse, respectively. The corresponding equation for each side of the polygon (i.e., each line that cuts the semi ellipse) is obtained as follows:

$$\frac{P_{it} - P_i^{max} \cos \gamma_k}{Q_{it} - Q_i^{max} \sin \gamma_k} = \frac{P_i^{max} \cos \gamma_{k+1} - P_i^{max} \cos \gamma_k}{Q_i^{max} \sin \gamma_{k+1} - Q_i^{max} \sin \gamma_k}, \quad \forall i, \forall t, \forall k, \quad (14)$$

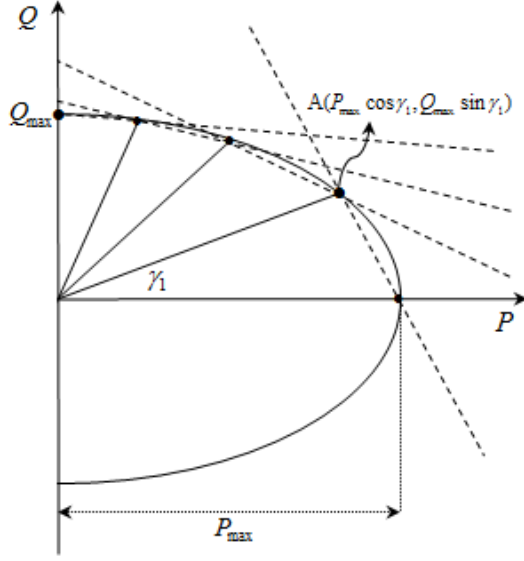


Fig. 1. Inner polyhedral linearization of the generation capacity curve.

therefore, we have,

$$Q_{it} - \left( \frac{Q_i^{max} (\sin \gamma_{k+1} - \sin \gamma_k)}{P_i^{max} (\cos \gamma_{k+1} - \cos \gamma_k)} \right) P_{it} \leq Q_i^{max} \left( \sin \gamma_k - \frac{\cos \gamma_k (\sin \gamma_{k+1} - \sin \gamma_k)}{\cos \gamma_{k+1} - \cos \gamma_k} \right), \quad \forall i, \forall t, \forall k, \quad (15)$$

Due to the symmetric shape of semi ellipse, the following constraints are imposed for the inner linear approximation of their lower half:

$$Q_{it} + \left( \frac{Q_i^{max} (\sin \gamma_{k+1} - \sin \gamma_k)}{P_i^{max} (\cos \gamma_{k+1} - \cos \gamma_k)} \right) P_{it} \geq -Q_i^{max} \left( \sin \gamma_k - \frac{\cos \gamma_k (\sin \gamma_{k+1} - \sin \gamma_k)}{\cos \gamma_{k+1} - \cos \gamma_k} \right), \quad \forall i, \forall t, \forall k, \quad (16)$$

#### D. Transmission Outage and Capacity Constraints

The active and reactive power flows in each transmission line are bounded to the maximum and minimum capacity. If a transmission line or any of the substations at the two ends of the transmission line are on outage, the line will be offline, hence the associated active and reactive power flows are set to zero. Binary variable  $w_{lt}$  is equal to 0, if the line  $l$  is offline at time  $t$  due to damage from disaster; otherwise, it is equal to 1. Therefore,

$$-PL_l^{max} w_{lt} \leq PL_{lt} \leq PL_l^{max} w_{lt}, \quad \forall l, \forall t, \quad (17)$$

$$-QL_l^{max} w_{lt} \leq QL_{lt} \leq QL_l^{max} w_{lt}, \quad \forall l, \forall t. \quad (18)$$

On the other hand, if any of the connected substations on two sides of the transmission line are damaged and are not restored by time  $t$  (i.e.,  $z_{bt}=0$ ), the associated active and reactive power flows in transmission line  $l$  will be equal to zero as shown in (19)-(22).

$$-M \sum_b \beta_{lb}^{from} z_{bt} \leq PL_{lt} \leq M \sum_b \beta_{lb}^{from} z_{bt}, \quad \forall l, \forall t, \quad (19)$$

$$-M \sum_b |\beta_{lb}^{to}| z_{bt} \leq PL_{lt} \leq M \sum_b |\beta_{lb}^{to}| z_{bt}, \quad \forall l, \forall t, \quad (20)$$

$$-M \sum_b \beta_{lb}^{from} z_{bt} \leq QL_{lt} \leq M \sum_b \beta_{lb}^{from} z_{bt}, \quad \forall l, \forall t, \quad (21)$$

$$-M \sum_b |\beta_{lb}^{to}| z_{bt} \leq QL_{lt} \leq M \sum_b |\beta_{lb}^{to}| z_{bt}, \quad \forall l, \forall t, \quad (22)$$

where  $\beta^{from}$  includes all positive elements of the bus-line incidence matrix and  $\beta^{to}$  includes all negative elements of the bus-line incidence matrix. Finally, the transmission line apparent power flow constraint holds as follows:

$$PL_{lt}^2 + QL_{lt}^2 \leq SL_{lt}^2, \quad \forall l, \forall t, \quad (23)$$

which is the equation of an ellipse, similar to generation capability curve. Using the same approach that was proposed for generation capability curve, the apparent power flow capacity can be linearized.

#### E. Voltage Control

In order to maintain the system stability, all voltages must be within the minimum and maximum limits as

$$V_{bt}^{min} \leq V_{bt} \leq V_{bt}^{max}, \quad \forall b, \forall t. \quad (24)$$

It is assumed that the magnitude of voltage at each node can be obtained from the AC based-point solution, i.e.,  $\tilde{V}_{bt}$ . It is also assumed that approximation of  $\sin(\delta_{nt} - \delta_{mt}) \approx \delta_{nt} - \delta_{mt}$  for small phase angle difference is accurate [2]. Based on these assumptions, the active and reactive power flows at each line are approximated as

$$PL_{lt} = |\tilde{V}_{nt}|^2 g_l - |\tilde{V}_{nt}| |\tilde{V}_{mt}| \left( g_l \widehat{\cos}_{nmt} + b_l (\delta_{nt} - \delta_{mt}) \right), \quad \forall \langle m, n \rangle \in N_l, \forall t, \quad (25)$$

$$QL_{lt} = -|\tilde{V}_{nt}|^2 b_l - |\tilde{V}_{nt}| |\tilde{V}_{mt}| \left( g_l (\delta_{nt} - \delta_{mt}) - b_l \widehat{\cos}_{nmt} \right), \quad \forall \langle m, n \rangle \in N_l, \forall t. \quad (26)$$

As shown, a linear system of equations are formed in (25)-(26). The cosine function of voltage angle difference between nodes  $m$  and  $n$  is considered as a continuous variable  $\widehat{\cos}_{nmt}$ . We can also consider  $\sin(\delta_{nt} - \delta_{mt}) \approx \delta_{nt} - \delta_{mt}$  as another continuous variable  $\widehat{\sin}_{nmt}$ , and add a constraint to the model

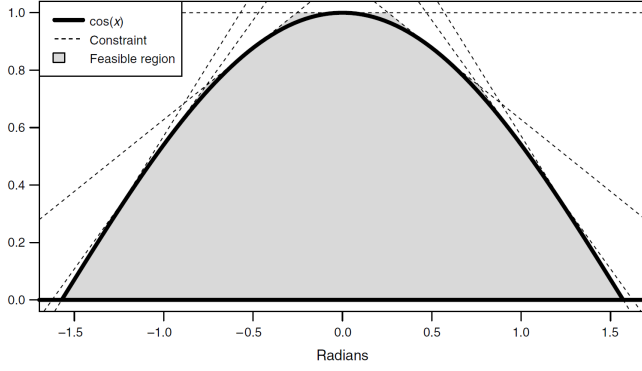


Fig. 2. A polyhedral relaxation of cosine using seven inequalities [2].

in order to control and approximate the voltage angle difference, as follows:

$$\widehat{\sin}_{nmt}^2 + \widehat{\cos}_{nmt}^2 = 1, \quad \forall \langle m, n \rangle \in N_l, \forall t, \quad (27)$$

which is equation for a circle. Using the same manner as generation capability curve and apparent power flow capacity, this constraint can be approximated as a polygon. An alternative approach is a polyhedral relaxation of the cosine function by constraining it to a set of hyperplane tangents proposed in [2]. As shown in Fig. 2, the tangent line to the cosine function in a given point  $a$  was defined in [2] by

$$y = -\sin(a)(x - a) + \cos(a) \quad \forall a \in (-\pi/2, \pi/2). \quad (28)$$

By evenly spacing the phase angle difference domain into  $h$  hyperplanes, the distance  $d$  between tangent points is obtained by  $d = \pi/(h + 1)$ . In [2] the summation of  $\widehat{\cos}_{nmt}$  was maximized in the objective function, and the following linear constraints were imposed to construct the polyhedral relaxation

$$\begin{aligned} \widehat{\cos}_{nmt} \leq & -\sin(jd - \pi/2)(\delta_{nt} - \delta_{mt} - jd + \pi/2) \\ & + \cos(jd - \pi/2), \quad \forall j \in \{1, 2, \dots, h\}, \forall \langle m, n \rangle \in N_l, \forall t. \end{aligned} \quad (29)$$

However, maximizing summation of  $\widehat{\cos}_{nmt}$  in objective function requires to solve a bi-objective problem which is not always desirable. Beside, the small range of cosine function's value compared to the much larger scale of other terms in the objective function can be problematic.

#### F. Restoration Resource Modeling

Constraints (30)-(31) present the relationships among binary outage variables  $w_{lt}$  and  $z_{bt}$  with repair decision variables  $v_{lt}$  and  $u_{bt}$ , respectively. If the transmission line  $l$  at time  $t$  is on outage, the binary variable  $w_{lt}$  which represents the line outage state would be equal to zero. Once it is repaired, the value of  $w_{lt}$  becomes 1, and remains the same up to the end of the outage management horizon.  $v_{lt}$  is the repair decision variable of transmission line  $l$ , in a sense that, when the line  $l$  is under repair at time  $t$ , the  $v_{lt}$  takes the value of 1, otherwise it is 0. In the same way,  $z_{bt}$  is the binary outage variable for substation

$b$ , which is equal to 0 when substation  $b$  at time  $t$  is on outage; Once it is repaired the value of  $w_{lt}$  becomes 1, and remains the same up to the end of the outage management horizon.  $u_{bt}$  is the decision variable for maintenance of substation  $b$ , which takes the value of 1, when the substation is under repair, otherwise it is equal to 0.

$$0 \leq w_{lt} - \left( \sum_{k=1}^t v_{lk} - TTR_l + 0.5 \right) / M \leq 1 \quad \forall l, \forall t, \quad (30)$$

$$0 \leq z_{bt} - \left( \sum_{k=1}^t u_{bk} - TTR_b + 0.5 \right) / M \leq 1 \quad \forall b, \forall t. \quad (31)$$

Constraint (32) represents the time that a damaged generation unit comes back to the system after repair. As earlier described, the utility company has no control over the restoration of generating units. However, the generating unit repair time is collected by the transmission company, and is incorporated in the restoration scheduling and coordination.

$$y_{it} = 0 \text{ if } t \leq TTR_i; \text{ otherwise } y_{it} = 1, \quad \forall i, \forall t. \quad (32)$$

However, since *if-then* constraint is not allowed in linear programming, constraint (32) is formulated as

$$t - My_{it} \leq TTR_i, \quad \forall i, \forall t, \quad (33)$$

$$My_{it} \leq TTR_i, \quad \forall i, \forall t = 0, 1, \dots, TTR_i. \quad (34)$$

Each damaged transmission line or substation should receive the required time and resources to be restored. In this model, it is assumed that once the restoration operation on a particular component is started, it should be continued for a duration of at least time to repair (TTR) of the component. Constraints (35)-(36) guarantee that enough time and resources are allocated to each damaged component to be repaired. Moreover, these constraints eliminate partial repair operation on each damaged component.

$$\sum_{k=t}^{t+TTR_l-1} v_{lk} \geq TTR_l(v_{lt} - v_{l(t-1)}), \quad \forall l, \forall t, \quad (35)$$

$$\sum_{k=t}^{t+TTR_b-1} u_{bk} \geq TTR_b(u_{bt} - u_{b(t-1)}), \quad \forall b, \forall t. \quad (36)$$

Restoration resource limitation is modeled as follows:

$$R_{lt} \geq R_l^{min} v_{lt}, \quad \forall l, \forall t, \quad (37)$$

$$R_{bt} \geq R_b^{min} u_{bt}, \quad \forall b, \forall t, \quad (38)$$

$$\sum_l R_{lt} + \sum_b R_{bt} \leq R_t^{max}, \quad \forall t. \quad (39)$$

where (37)-(38) represent the required number of crews to be allocated at time  $t$  to line  $l$ , and substation  $b$ , respectively; and (39) indicates the maximum number of available crew that can be allocated during each hour.

#### IV. NUMERICAL EXAMPLE

The standard IEEE 118-bus test system is used for numerical analysis of the proposed restoration model. It is assumed that four substations, five transmission lines, and three generation units have damaged, and are required to be restored. Among damaged substations, three of them (i.e., B2, B3, and B11) are load buses, feeding their downstream distribution lines. Table I indicates the time it takes from the beginning of the restoration process to repair and restore each damaged generating unit. Second column of Tables II and III show the estimated repair duration of damaged substations (along with their downstream distribution lines) and damaged transmission lines, respectively.

The value of lost load is assumed to be \$100/kWh for critical loads, e.g. medical centers, \$37.06/kWh for commercial loads, and \$1.1/kWh for residential loads as they were used in [13]. Among damaged buses, B4 is connected to a critical load, B11 is connected to a commercial load, and the rest of the substations are connected to residential loads. Without loss of generality, the value for voltage magnitude in all nodes are considered to be 1 p.u. Each hour of restoration operation on a substation requires the crew size of 15 people, while it requires the crew of 20 people per hour for each transmission line. The total number of available crew is limited to 80 people per hour. The restoration planning horizon is set to be 120 hours.

TABLE I  
DAMAGED GENERATION UNITS AND TIME TO REPAIRS

Unit Number	Time to Repair
G2	15
G3	24
G5	12

TABLE II  
DAMAGED SUBSTATIONS, TTRS, AND RESTORATION SCHEDULES

Bus Number	Time to Repair	AC Schedule	DC Schedule
B2	14	1-14	8-21
B4	18	1-18	1-18
B8	7	85-91	1-7
B11	22	1-22	1-22

TABLE III  
DAMAGED TRANSMISSION LINES, TTRS, AND RESTORATION SCHEDULES

Line Number	Time to Repair	AC Schedule	DC Schedule
L1	15	19-33	19-33
L2	10	1-10	1-10
L10	20	11-30	22-41
L14	17	15-31	22-38
L16	18	19-36	16-33

The proposed AC model for the described system data is solved using CPLEX 12.1 with 0.01 relative optimality gap. In addition, by removing AC power flow constraints from the model, a DC version of the proposed restoration model is solved. The third and fourth columns of Tables II and III show the optimal schedules for restoration of the system using AC and DC restoration models, respectively. As

shown, the loading buses are restored with higher priority in both models. However, bus B8 which is not a loading bus and does not play a critical role in transmission dynamics, is last component of the system to be recovered in AC model. The restoration of transmission lines varies based on their importance on recovering the system load. The total load interruption cost of the system in AC and DC models are \$9,543,726 and \$9,479,807, respectively. Intuitively, the slightly higher interruption cost of AC model results from satisfying the AC power flow constraints in restoration process.

#### V. CONCLUSIONS

A model for restoration of transmission network after natural disasters was proposed. The AC power flow constraints were considered in the model. Mixed-integer programming was used to formulate the problem. The induced nonlinearity due to AC constraints were linearized using polyhedral approximation. The proposed model was tested on the standard IEEE 118-bus test system. The extended numerical analysis of the proposed model is left for future work.

#### REFERENCES

- [1] B. Stott, J. Jardim, and O. Alsa, "DC power flow revisited," *Power Systems, IEEE Transactions on*, vol. 24, no. 3, pp. 1290-1300, Aug. 2009.
- [2] C. Coffrin and P. V. Hentenryck, "A linear-programming approximation of AC power flows," *INFORMS Journal on Computing*, vol. 26, no. 4, May 2014.
- [3] J. A. Taylor and F. S. Hover, "Linear relaxations for transmission system planning," *Power Systems, IEEE Transactions on*, vol. 26, no. 4, pp. 2533-2538, Nov. 2011.
- [4] S. H. Low, "Convex relaxation of optimal power flow, Part II: Exactness," *Control of Network Systems, IEEE Transactions on*, vol. 1, no. 2, pp. 177-189, Jun. 2014.
- [5] C. W. Tan, D. W. Cai, and X. Lou, "Resistive network optimal power flow: uniqueness and algorithms," *Power Systems, IEEE Transactions on*, vol. 30, no. 1, pp. 263-273, Jan. 2015.
- [6] S. Guha, A. Moss, J. S. Naor, and B. Schieber, "Efficient recovery from power outage," in *Proc. 31th ACM Symp. on Theory of Comp.*, Atlanta, May 1999.
- [7] A. C. Chien, "Optimized recovery of damaged electrical power grids," M.S. thesis, Dept. Operations Res., Naval Postgr. School, Monterey, CA, 2006.
- [8] S. Thibaux, C. Coffrin, H. Hijazi, and J. Slaney, "Planning with MIP for supply restoration in power distribution systems," in *Proc. of 23rd Int. joint Conf. on Artif. Intell.*, Beijing, China, Aug. 2013.
- [9] T. C. Matisziw, A. T. Murray, and T. H. Grubestic, "Strategic network restoration," *Netw. and Spat. Econ.*, vol. 10, no. 3, pp. 345-361, Sep. 2010.
- [10] S. G. Nurre, B. Cavdaroglu, J. E. Mitchell, T. C. Sharkey, and W. A. Wallace, "Restoring infrastructure systems: An integrated network design and scheduling (INDS) problem," *Euro. Jour. of Operat. Res.*, vol. 223, no. 3, pp. 794-806, Dec. 2012.
- [11] P. V. Hentenryck, C. Coffrin, and R. Bent, "Vehicle routing for the last mile of power system restoration," in *Proc. 17th Power Sys. Comp. Conf.*, Stockholm, Sweden, Aug. 2011.
- [12] B. Simon, C. Coffrin, and P. V. Hentenryck "Randomized adaptive vehicle decomposition for large-scale power restorations," in *Proc. 9th Int. Conf. on Integration of AI and OR Tech. in Cons. Prog. for Combin. Optim. Prob.*, Nantes, France, Jun. 2012.
- [13] A. Arab, A. Khodaei, S. K. Khator, K. Ding, V. A. Emesih, and Z. Han, "Stochastic pre-hurricane restoration planning for electric power systems infrastructure," *Smart Grid, IEEE Transactions on*, vol. 6, no. 2, pp. 1046-1054, Jan. 2015.
- [14] A. Arab, A. Khodaei, Z. Han, and S. K. Khator, "Proactive recovery of electric power assets for resiliency enhancement," *Access, IEEE*, vol. 3, pp. 99-109, Feb. 2015.